A graph is a mathematical construct that represents information about connections between pairs of objects. As a result, graphs are widely used as a modeling tool in engineering, social sciences, and other fields. The paper written by Leonhard Euler in 1736 on the Seven Bridges of Königsberg is often regarded as the starting point of graph theory; and we have come a long way since. This talk will survey a few classical problems in graph theory, and explore their relationship to the fields of research that are active today. In particular, we will discuss Ramsey theory, graph coloring, perfect graphs, as well as some more recent research directions.