WAITING TIME PHENOMENON FOR THE THIN-FILM EQUATION: SHARP CRITERIA IN TERMS OF THE MASS OF THE INITIAL DATA

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The thin-film equation, $\partial_t u + \partial_x \left(u^n \partial^3_{xxx} u\right) = 0$, is a degenerate fourth-order parabolic equation which arises in modelling the evolution of capillarity driven thin viscous films and spreading droplets on a solid substrate. Its degeneracy allows for the construction of mass-conserving nonnegativity-preserving solutions with the property of finite speed of propagation. Moreover, if the initial data is "flat enough" at the free boundary, a finite *waiting time* exists during which the interface does not move forward. In the regime of weak slippage, i.e. $n \in (2,3)$, the upper bound on the waiting time at a point x_0 of the free boundary is given in terms of the negative n-th power of the quantity

$$\left(\sup_{r>0} r^{-\frac{4}{n}(\alpha+1)} \fint_{B(x_0,r)} u^{1+\alpha}(0,x)\ \mathrm{d}x\right)^{\frac{1}{1+\alpha}},$$

for some $\alpha < 0$, and the lower bound on the waiting time at x_0 is given in terms of the negative n-th power of the quantity

$$\left(\sup_{r>0} r^{-2\left(\frac{4}{n}-1\right)} \fint_{B(x_0,r)} \left|\partial_x u(0,x)\right|^2\ \mathrm{d}x\right)^{\frac{1}{2}}.$$

The aim of this talk is to bridge this gap between the known sufficient and necessary conditions and to present new sharp criteria which only depend on the mass distribution of the initial data, i.e., more precisely, on the negative n-th power of the quantity

$$\sup_{r>0} r^{-\frac{4}{n}} \fint_{B(x_0,r)} u(0,x)\ \mathrm{d}x.$$

The key ingredients of the proofs are the following:

(1) a monotonicity formula introduced in [Fischer, 2014 & 2016];
(2) an iteration technique based on a localized mass estimate and a time-weighted localized energy estimate.

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