In 2017 Ghosh and Sarnak predicted how often the integral Hasse principle should fail in the family of Markoff surfaces. They were able to obtain a lower bound for this quantity but its magnitude is still far away from the expectation. In this talk we will explain what can be achieved for this family using tools from Arithmetic geometry developed for studying failures of local-to-global principles. In particular, we will explain how to get sharp upper and lower bounds for the number of failures of the Hasse principle for integral points explained by the Brauer-Manin obstruction. Moreover, we will give a lower bound for the number of failures which are not explained by the Brauer-Manin obstruction. This talk is based on a join work with Dan Loughran.