The classical work of Pitman in probability theory establishes a surprising link between the Brownian motion in dimensions one and three. This relation was interpreted by Biane-Bougerol-O’Connell in terms of the Duistermaat-Heckman measure from symplectic geometry.

We generalize these constructions for the case of Brownian motion on curved three-dimensional spaces: the 3-sphere and the hyperbolic space. The case of the hyperbolic space is intimately related to the quantum group $U_q(sl(2))$. The method is a combination of analytic results and numerical experiments which allowed to rule out some of the scenarios.