In 1872 Felix Klein introduced a new approach to view geometry as the study of symmetries. This approach has been very influential both in mathematics as well as in physics. It allows to study different geometric structures in a unified framework, and to investigate the interplay between topological manifolds and their geometric realizations in a new way. In the past twenty years several new developments led to a revival of these ideas and to a paradigm shift in the study of discrete symmetry groups acting on spaces of non-positive curvature. In my talk I will describe the general framework, highlight some of the new developments and discuss selected results on the geometry and dynamics of such group actions.