In this talk we will discuss various topological aspects of random graphs. How does the genus of a uniform random graph change as the number of edges increases? How does a topological constraint (such as imposing an upper bound on the genus) influence the structure of a random graph (such as the order of the largest component, the length of the shortest and longest cycles)?