



## Mathematics and CS Seminar

# The isometry group of Wasserstein spaces: the Hilbertian case

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**Host: Laszlo Erdős**

I will report on the most recent step of our systematic study of Wasserstein isometries, which is joint work with Gyorgy Pal Geher (U Reading) and Tamas Titkos (Renyi Inst., Budapest). Now we consider Wasserstein spaces over a separable real Hilbert space  $H$  and describe the isometries for every positive finite parameter  $p$ . The quadratic case ( $p=2$ ) turns out to be an infinite-dimensional analogon of Kloeckner's result on the isometries of  $W_2(\mathbb{R}^n)$  from 2010, which says that  $W_2(\mathbb{R}^n)$  admits non-trivial isometries as well as trivial ones (which are governed by isometries of the underlying space). For  $p \neq 2$ , we use a two-step argument. First, we give a metric characterization of Dirac masses and deduce that they are invariant under Wasserstein isometries (modulo trivial isometries). This metric characterization is essentially different for concave cost ( $p < 1$ ) and for convex cost ( $p \geq 1$ ). Then we introduce a quantity which we call the Wasserstein potential of a measure and which is invariant under Wasserstein isometries. We show that the potential function completely determines the measure for every non-even positive  $p$ , and hence we deduce isometric rigidity, which means that  $\text{Isom}(W_p(H)) = \text{Isom}(H)$ . For  $p=4,6,8,\dots$  we prove isometric rigidity, although in this case, the Wasserstein potential does not carry enough information to recover the measure.

If time allows, I will demonstrate the efficiency of the potential function method on different underlying spaces (including spheres, tori, and projective planes) as well.

**Thursday, October 22, 2020 04:30pm - 05:30pm**

IST Austria Campus online via Zoom



This invitation is valid as a ticket for the IST Shuttle from and to Heiligenstadt Station. Please find a schedule of the IST Shuttle on our webpage: <https://ist.ac.at/en/campus/how-to-get-here/> The IST Shuttle bus is marked IST Shuttle (#142) and has the Institute Logo printed on the side.