In this talk, we will discuss the quantitative Tracy-Widom law for the largest eigenvalue of Wigner matrices, as well as sample covariance matrices. More precisely, we will prove that the fluctuations of the largest eigenvalue of a Wigner matrix of size $N$ converge to its Tracy-Widom limit at a rate nearly $N^{-1/3}$, as $N$ tends to infinity. Our result follows from a quantitative Green function comparison theorem, originally introduced by Erdos, Yau and Yin to prove edge universality, on a finer spectral parameter scale with improved error estimates. The proof relies on the continuous Green function flow induced by a matrix-valued Ornstein-Uhlenbeck process. Precise estimates on leading contributions from the third and fourth order moments of the matrix entries are obtained using iterative cumulant expansions and recursive comparisons for correlation functions, along with uniform convergence estimates for correlation kernels of the Gaussian ensembles. This is joint work with Kevin Schnelli.