We give a presentation of the n-dimensional oriented cobordism category Cob \_n with generators corresponding to diffeomorphisms and surgeries along framed spheres, and a complete set of relations. Hence, given a functor F from the category of smooth oriented manifolds and diffeomorphisms to an arbitrary category C, and morphisms induced by surgeries along framed spheres, we obtain a necessary and sufficient set of relations these have to satisfy to extend to a functor from Cob \_n to C. If C is symmetric and monoidal, then we also characterize when the extension is a TQFT. This framework is well-suited to defining natural cobordism maps in Heegaard Floer homology. Finally, we use it to classify (2+1)-dimensional TQFTs in terms of J-algebras, a new algebraic structure that consists of a split graded involutive nearly Frobenius algebra endowed with a certain mapping class group representation.